arXiv:hep-ph/9309268v1 15 Sep 1993

THE STANDARD MODEL AT LOW ENERGIES*

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Abstract

The hadronic sector of the standard model at low energies is described by a non–decoupling effective field theory, chiral perturbation theory. An introduction is given to the construction of effective chiral Lagrangians, both in the purely mesonic sector and with inclusion of baryons. The connection between the relativistic formulation and the heavy baryon approach to chiral perturbation theory with baryons is reviewed.

Lectures given at the
6th Indian–Summer School on Intermediate Energy Physics
Interaction in Hadronic Systems
Prague, August 25 - 31, 1993
To appear in the Proceedings (Czech. J. Phys.)

Contents

1   EFFECTIVE FIELD THEORIES

Until some 25 years ago, quantum field theory as a tool for making experimentally testable predictions in particle physics seemed to be restricted to quantum electrodynamics. The S–matrix approach was widely considered to be a more promising method for the strong and even the weak interactions. The development and the impressive success of the standard model of strong and electroweak interactions have led to a grand revival of quantum field theories. In the more recent past, so-called effective field theories (EFT) have been used more and more frequently. In a certain sense, EFT can be regarded as a synthesis of S–matrix theory and quantum field theory [1]. Phrased in a different manner, Lagrangian quantum field theory offers an explicit framework for implementing systematically all the general axioms of S–matrix theory like analyticity, unitarity and cluster decomposition for a finite number of degrees of freedom [2] (the revenge of S–matrix theory according to Weinberg [1]). Even our present "fundamental" quantum field theories are in all likeliness just EFT below some energy scale. All we know about this scale is that it must be larger than the presently available energies.

The notion of EFT is only practicable if we have, in addition to the general axioms of S–matrix theory, some other criteria at our disposal to constrain the corresponding effective Lagrangians. These criteria are the symmetries we abstract from the "fundamental" underlying theory, Lorentz invariance being the

most obvious one. In this context, it is useful to distinguish between two types of EFT.

A. Decoupling EFT

For energies below some scale Λ, all heavy (with respect to Λ) degrees of freedom are integrated out leaving only the light degrees of freedom in the effective theory. No light particles are generated in the transition from the fundamental to the effective level. The effective Lagrangian has the general form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{d \leq 4} + \sum_{d > 4} \frac{1}{\Lambda^{d-4}} \sum_{i_d} g_{i_d} O_{i_d} \tag{1.1}$$

where $\mathcal{L}_{d \leq 4}$ contains the potentially renormalizable terms with operator dimension $d \leq 4$, the g_{i_d} are dimensionless coupling constants, expected to be at most of $O(1)$, and the O_{i_d} are monomials in the light fields with operator dimension d. At energies much below Λ, it may be a good approximation to work with the renormalizable Lagrangian $\mathcal{L}_{d \leq 4}$ only, because the corrections due to the non–renormalizable parts ($d > 4$) will be suppressed by powers of E/Λ. In such cases, $\mathcal{L}_{d \leq 4}$ can be regarded as the "fundamental" Lagrangian at low energies.

There are several well–known examples of decoupling EFT.

a. QED for $E \ll m_e$
   For photon energies much smaller than the electron mass, the electrons can be integrated out to yield the Euler–Heisenberg Lagrangian for light-by-light scattering [3]

$$\mathcal{L}_{\text{EH}} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2) + \frac{e^4}{360\pi^2 m_e^4} \left[(\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E} \cdot \vec{B})^2 \right] + \dots \tag{1.2}$$

   The coefficient in front of the $d = 8$ interaction term can be calculated explicitly from the box diagram, because the "fundamental" theory QED can be treated perturbatively. Here, the renormalizable part is only the kinetic term.

b. Weak interactions for $E \ll M_W$
   At low energies, the weak interactions reduce to the Fermi theory including the neutral current interactions. After integrating out the W and Z bosons, the weak interactions appear first in the $d = 6$ terms.

c. The standard model for $E \ll 1$ TeV
   There is no reason to believe that the standard model will be valid down to arbitrarily small distances. Instead, it should be regarded as the EFT of some more fundamental theory at higher energies (SUSY, grand unification,

superstrings,...). At presently available energies, however, all experimental evidence is still in agreement with the standard model Lagrangian being "fundamental".

B. Non–decoupling EFT

In the transition from the fundamental to the effective level, a phase transition occurs via the spontaneous breakdown of some symmetry generating light ($M \ll \Lambda$) pseudo–Goldstone bosons. Since a spontaneously broken symmetry relates processes with different numbers of Goldstone bosons, a split of the effective Lagrangian like in Eq. (1.1) into renormalizable ($d \leq 4$) and non–renormalizable ($d > 4$) parts becomes meaningless. The effective Lagrangian of a non–decoupling EFT is intrinsically non–renormalizable. As the developments of the last decade have shown [2, 4, 5], this does not prevent such EFT from being perfectly consistent quantum field theories. Instead of the operator dimension like in the effective Lagrangian (1.1), it is the number of derivatives of the fields that distinguishes successive terms in the Lagrangian.

The general structure of effective Lagrangians with spontaneously broken symmetries is to a large extent independent of the specific physical realization. This can already be seen from two examples in particle physics.

a. The standard model without Higgs bosons
   Even if there is no explicit Higgs boson or if its mass is bigger than about 1 TeV, the gauge symmetry $SU(2) \times U(1)$ can be spontaneously broken to $U(1)_{\text{em}}$ (see, e.g., Ref. [6] for a recent review of the heavy Higgs scenario). Since it is a local symmetry that is spontaneously broken, the Goldstone bosons are not physical particles, but they become the longitudinal components of the W and Z bosons. As a manifestation of the universality of Goldstone boson interactions, the scattering of longitudinal vector bosons is in first approximation completely analogous to $\pi\pi$ scattering.

b. The standard model for $E < 1$ GeV
   At low energies, the relevant degrees of freedom of the standard model are not quarks and gluons, but the pseudoscalar mesons (and eventually other hadrons), the pseudo-Goldstone bosons of spontaneously broken chiral symmetry. The standard model in the hadronic sector is described by a non–decoupling EFT called chiral perturbation theory (CHPT).

The applications of EFT can again be classified in two groups:

● If the fundamental theory is unknown, EFT can be used to parametrize the effects of new physics. Examples are the heavy Higgs scenario and the search for effects of grand unification.

- Even if the fundamental theory is known, EFT can be useful. Either the full structure of the underlying theory is not needed as for light-by-light scattering, or the fundamental theory is not applicable with reliable methods in the region of interest. The latter case applies to QCD in the confinement regime where CHPT takes over.

In Sect. 2, a short introduction to the basic ingredients of CHPT will be given. The effective chiral Lagrangian for the pseudoscalar mesons up to $O(p^4)$ will be discussed in Sect. 3. Some recent applications of CHPT in the mesonic sector are briefly summarized. Finally, in Sect. 4 both the relativistic and the so–called non–relativistic formulation of baryon CHPT are reviewed. As an interesting application, neutral pion photoproduction off nucleons at threshold is treated in some detail.

In addition to the references given during the course of the lectures, more detailed material both on the structure and on applications of CHPT can be found in Refs. [7 – 15].

2 CHIRAL PERTURBATION THEORY

CHPT is the EFT of the standard model at low energies in the hadronic sector. Since as an EFT it contains **all** terms allowed by the symmetries of the underlying theory [2], it should not be viewed as a particular model for hadrons, but rather as a direct consequence of the standard model itself. The main assumptions of CHPT are:

- The masses of the light quarks u, d and possibly s can be treated as perturbations.

- In the limit of zero quark masses, the resulting chiral symmetry is spontaneously broken to its vectorial subgroup, isospin ($n = 2$) or flavour $SU(3)$ ($n = 3$) for n massless quarks. The resulting Goldstone bosons are the pseudoscalar mesons.

CHPT is a systematic expansion around the chiral limit [2, 4, 5]. As an EFT, it is a low–energy expansion in momenta (and pseudoscalar meson masses). Here are some of its prominent features:

- It is a well–defined quantum field theory even though it is non–renormalizable.

- There is no problem of double–counting of degrees of freedom: the effective chiral Lagrangian contains only hadron fields, but neither quarks nor gluons.

- The softly broken chiral symmetry and electromagnetic gauge invariance are manifest. The corresponding Ward identities are automatically satisfied.

- CHPT is ideally suited for the physics of pseudoscalar mesons, but other hadrons can be coupled in a chiral invariant manner (cf. Sect. 4 for the inclusion of baryons).

- The chiral anomaly [16] is unambiguously incorporated at the hadronic level [17].

- Although the structure of Green functions and amplitudes is calculable in CHPT, they contain parameters which are not restricted by the symmetries of the standard model. At each order in the chiral expansion, those low–energy constants must be determined either from phenomenology or with additional model–dependent assumptions.

2.1 Non–Linear Realization of Chiral Symmetry

The QCD Lagrangian for n massless quarks $q = (u, d, \ldots)$

$$
\begin{aligned}
\mathcal{L}^0_{\mathrm{QCD}} &= \overline{q} i \gamma^\mu \left(\partial_\mu + i g_s \frac{\lambda_\alpha}{2} G^\alpha_\mu \right) q - \frac{1}{4} G^\alpha_{\mu\nu} G^{\alpha\mu\nu} + \mathcal{L}_{\text{heavy quarks}} \qquad (2.1) \\
&= \overline{q_L} i \not{D} q_L + \overline{q_R} i \not{D} q_R + \ldots \\
q_{R,L} &= \frac{1}{2}(1 \pm \gamma_5) q
\end{aligned}
$$

has a global symmetry

$$
\underbrace{SU(n)_L \times SU(n)_R}_{\text{chiral group } G} \times U(1)_V \times U(1)_A .
$$

All theoretical and phenomenological evidence suggests that the chiral group G is spontaneously broken to the vectorial subgroup $SU(n)_V$. The axial generators of G are non–linearly realized and there are $n^2 - 1$ massless pseudoscalar Goldstone bosons.

There is a well–known procedure [18] how to realize a spontaneously broken symmetry on quantum fields. In the special case of chiral symmetry with its parity transformation, the Goldstone fields can be collected in a unitary matrix field $U(\phi)$ transforming as

$$
U(\phi) \xrightarrow{g} g_R U(\phi) g_L^{-1} \qquad g = (g_L, g_R) \in G \qquad (2.2)
$$

under chiral rotations. There are different parametrizations of $U(\phi)$ corresponding to different choices of coordinates for the chiral coset space $SU(n)_L \times$

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$SU(n)_R/SU(n)_V$. A convenient choice is the exponential parametrization (for $n = 3$)

$$U(\phi) = \exp\left(i\lambda_a\phi^a/F\right) \qquad \Phi = \frac{1}{\sqrt{2}}\lambda_a\phi^a = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\dfrac{2\eta_8}{\sqrt{6}} \end{pmatrix} , \tag{2.3}$$

where F will turn out to be the pion decay constant in the chiral limit. A more basic quantity from a geometrical point of view is another matrix field $u(\phi)$ which in the standard choice of coset coordinates is just the square root of $U(\phi)$ [18]. Its chiral transformation

$$u(\phi) \xrightarrow{g} g_R u(\phi)h(g,\phi)^{-1} = h(g,\phi)u(\phi)g_L^{-1} \tag{2.4}$$

introduces the so–called compensator field $h(g,\phi)$ representing an element of the conserved subgroup $SU(n)_V$. For $g \in SU(n)_V$, i.e. for $g_L = g_R$, the compensator $h(g)$ is a usual unitary representation matrix, independent of the Goldstone fields ϕ (linear representation à la Wigner–Weyl). For a proper chiral transformation $(g_L \neq g_R)$, on the other hand, $h(g,\phi)$ does depend on ϕ (non–linear realization à la Nambu–Goldstone).

One may wonder why we should bother to introduce the matrix $u(\phi)$ with its concomitant compensator field, if we can construct the effective chiral Lagrangian with the square $U = u^2$ with its simpler transformation rule (2.2). In fact, in the mesonic sector the choice of representation is merely a matter of taste. Let us construct as an exercise the lowest–order non–trivial chiral invariant with Goldstone fields only. As a consequence of the Goldstone theorem, the only such invariant without derivatives is a trivial constant. Instead of the derivative $\partial_\mu U$ we can also consider the so–called vielbein field

$$u_\mu = i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger) = iu^\dagger\partial_\mu U u^\dagger , \tag{2.5}$$

which transforms as

$$u_\mu \xrightarrow{g} h u_\mu h^{-1} \tag{2.6}$$

under chiral transformations. Since the chiral symmetry is global for the time being, $\partial_\mu U$ transforms like U itself in Eq. (2.2). The unique chiral invariant of lowest order, $O(p^2)$, can now be written in two equivalent ways as ($\langle\ldots\rangle$ denotes the trace in n–dimensional flavour space)

$$\langle u_\mu u^\mu \rangle = \langle \partial_\mu U^\dagger \partial^\mu U \rangle , \tag{2.7}$$

which is nothing but the non–linear σ model up to a constant factor.

The representation involving the compensator field $h(g, \phi)$ is more convenient if we want to couple non–Goldstone degrees of freedom like meson resonances (Sect. 3) or baryons (Sect. 4) (see Ref. [7] for a discussion of different possibilities how to represent the baryons in chiral Lagrangians). Once we specify the transformation behaviour of (non–Goldstone) matter fields Ψ under the flavour group $SU(n)_V$, it is straightforward to construct a realization of the whole chiral group G [18]. Let us take the nucleon doublet ($n = 2$) Ψ as an example. Under isospin it transforms as

$$\Psi \xrightarrow{g} \Psi' = h(g)\Psi \qquad g \in SU(2)_V \tag{2.8}$$

where $h(g)$ is just the defining representation of $SU(2)$. For a general chiral transformation $g \in G$ one defines

$$\Psi \xrightarrow{g} \Psi' = h(g, \phi)\Psi \tag{2.9}$$

in terms of the compensator field $h(g, \phi)$ in the doublet representation [1]. It is easy to verify that (2.9) is a realization of G. However, since the transformation (2.9) is local via $h(g, \phi(x))$, the normal derivative $\partial_\mu \Psi$ does not transform covariantly. The necessary connection is given by

$$\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) \ . \tag{2.10}$$

The corresponding covariant derivative

$$\nabla_\mu \Psi = \partial_\mu \Psi + \Gamma_\mu \Psi \tag{2.11}$$

transforms indeed like Ψ itself. In this framework, the construction of chiral invariant Lagrangians is almost trivial. Similar to gauge theories, it boils down to replacing normal by covariant derivatives in $SU(n)_V$ invariant Lagrangians. Of course, the recipe is so simple because the Goldstone fields appear only in the matrix field $u(\phi)$.

3   MESONS

3.1   Effective Chiral Lagrangian

We first extend the QCD Lagrangian (2.1) by coupling the quarks to external hermitian matrix–valued fields v_μ, a_μ, s, p:

$$\mathcal{L} = \mathcal{L}_{\text{QCD}}^0 + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}(s - i\gamma_5 p)q \ . \tag{3.1}$$

The external field method offers several advantages [4, 5]. Its two most important features are:

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[1]Of course, the same procedure works for arbitrary representations of $SU(n)$.

- The electromagnetic and semileptonic weak interactions are automatically included with ($n = 3$ for the rest of this section)

$$r_\mu = v_\mu + a_\mu \;=\; -eQA_\mu \tag{3.2}$$
$$l_\mu = v_\mu - a_\mu \;=\; -eQA_\mu - \frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ T_+ + h.c.)$$

$$Q = \frac{1}{3}\mathrm{diag}(2,-1,-1)\,, \qquad T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where the V_{ij} are Kobayashi–Maskawa matrix elements.

- The method provides an elegant way to incorporate explicit chiral symmetry breaking through the quark masses. The physical Green functions (and S–matrix elements) are obtained as functional derivatives of a generating functional $Z[v,a,s,p]$ to be defined below at

$$v_\mu = a_\mu = p = 0\,,$$

but at
$$s = \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)\,. \tag{3.3}$$

The tremendous advantage is that we can calculate $Z[v,a,s,p]$ in a manifestly chiral invariant way until the very end.

The previous global chiral symmetry is now promoted to a local $SU(3)_L \times SU(3)_R$ symmetry

$$\begin{aligned}
q_{R,L} &\;\rightarrow\; g_{R,L} q_{R,L} \\
r_\mu &\;\rightarrow\; g_R r_\mu g_R^\dagger + i g_R \partial_\mu g_R^\dagger \\
l_\mu &\;\rightarrow\; g_L l_\mu g_L^\dagger + i g_L \partial_\mu g_L^\dagger \\
s + ip &\;\rightarrow\; g_R(s + ip) g_L^\dagger \\
g_{R,L} &\;\in\; SU(3)_{R,L}\,.
\end{aligned} \tag{3.4}$$

The local nature of the chiral symmetry requires the introduction of a covariant (now with respect to the external gauge fields v, a) derivative

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu \tag{3.5}$$

and of non–Abelian field strength tensors

$$\begin{aligned}
F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu] \\
F_L^{\mu\nu} &= \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu]\,.
\end{aligned} \tag{3.6}$$

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In the mesonic sector, the generating functional $Z[v, a, s, p]$ can be defined via

$$e^{iZ[v,a,s,p]} = N' < 0|Te^{i \int d^4 x \mathcal{L}}|0> = N \int [dU(\phi)] e^{i \int d^4 x \mathcal{L}_{\text{eff}}} \ , \qquad (3.7)$$

where the normalization constants N, N' ensure $Z[0, 0, 0, 0] = 0$. The first equation is the general definition of a generating functional of Green functions in terms of the fundamental Lagrangian (3.1). The content of the second equation is that we shall perform the actual calculations with the EFT of the standard model characterized by an effective chiral Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_M = \mathcal{L}_2 + \mathcal{L}_4 + \dots \ , \qquad (3.8)$$

written as an expansion in derivatives and external fields. The chiral counting rules are :

$$
\begin{array}{cc}
U & O(p^0) \\
D_\mu U, v_\mu, a_\mu & O(p) \\
s, p, F_{L,R}^{\mu\nu} & O(p^2)
\end{array}
\qquad (3.9)
$$

which are obvious except for s, p, to be explained shortly.

The lowest order (locally) chiral invariant Lagrangian

$$\mathcal{L}_2 = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle \qquad (3.10)$$

is the generalization of the non–linear σ model (2.7) in the presence of external fields with

$$\chi = 2B_0(s + ip) \ . \qquad (3.11)$$

The parameters F and B_0 are the only free constants at $O(p^2)$. From a calculation of the axial–vector current for the Lagrangian (3.10), one finds that F equals the pion decay constant to lowest order in CHPT :

$$F = F_\pi = 93.2 \text{ MeV} \ . \qquad (3.12)$$

The second constant B_0 is related to the quark condensate :

$$\langle 0|\overline{q_i}q_j|0\rangle = -F^2 B_0 \delta_{ij} \ . \qquad (3.13)$$

It sets the overall scale for the pseudoscalar meson masses at lowest order, e.g.

$$M_{\pi^+}^2 = (m_u + m_d)B_0 \ . \qquad (3.14)$$

The last equation explains why the scalar fields s and χ are counted as $O(p^2)$ in the chiral expansion. Since $B_0 \neq 0$ in the chiral limit, the squares of the

meson masses are proportional to the quark masses contained in s or χ. Having determined the two free constants F, B_0, one can now use the effective Lagrangian (3.10) or more precisely the generating functional $Z_2[v, a, s, p]$ to derive all the current algebra results for mesonic Green functions and amplitudes in terms of F and meson masses. The constant B_0 is hidden in the meson masses and will never appear explicitly.

The calculation of Green functions and amplitudes to next–to–leading order p^4 requires three steps [4, 5] :

- The general effective chiral Lagrangian $\mathcal{L}_4$ to be considered at tree level.

- The one–loop functional for the lowest–order Lagrangian $\mathcal{L}_2$ in (3.10).

- The Wess–Zumino–Witten functional [17] to account for the chiral anomaly [16].

The general chiral Lagrangian $\mathcal{L}_4(U, v, a, s, p)$ [5],

$$
\begin{aligned}
\mathcal{L}_4 \;=\; & L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \qquad\qquad (3.15) \\
& + L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \rangle + L_4 \langle D_\mu U^\dagger D^\mu U \rangle \langle \chi^\dagger U + \chi U^\dagger \rangle \\
& + L_5 \langle D_\mu U^\dagger D^\mu U (\chi^\dagger U + U^\dagger \chi) \rangle + L_6 \langle \chi^\dagger U + \chi U^\dagger \rangle^2 + L_7 \langle \chi^\dagger U - \chi U^\dagger \rangle^2 \\
& + L_8 \langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger \rangle - i L_9 \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \rangle \\
& + L_{10} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle + L_{11} \langle F_{R\mu\nu} F_R^{\mu\nu} + F_{L\mu\nu} F_L^{\mu\nu} \rangle + L_{12} \langle \chi^\dagger \chi \rangle \;,
\end{aligned}
$$

depends on 10 a priori arbitrary low–energy constants $L_1, \ldots, L_{10}$ (L_{11} and L_{12} are not directly accessible experimentally). These low–energy constants are the price we have to pay for using an EFT instead of the fundamental Lagrangian (3.1).

Before discussing the low–energy constants further, let us consider a general L–loop amplitude and determine its chiral dimension. In this section, all internal lines correspond to pseudoscalar mesons with propagators

$$
\frac{i}{k^2 - M_a^2} \qquad\qquad a = 1, \ldots, 8 \;. \qquad\qquad (3.16)
$$

A general diagram contains N_d^M vertices of $O(p^d)$ ($d = 2, 4, \ldots$) corresponding to the terms in the effective Lagrangian (3.8). After the loop integrations, the amplitude is a homogeneous function of the external momenta, the pseudoscalar masses M_a and the external fields. The degree of homogeneity is called the chiral dimension of the L–loop amplitude :

$$
D_L = 4L - 2I + \sum_d d N_d^M, \qquad\qquad (3.17)
$$

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where L is the number of loops and I the number of internal lines. Using the topological relation

$$L = I - \sum_d N_d^M + 1 \;, \tag{3.18}$$

valid for any simply connected diagram, we obtain [2]

$$D_L = 2L + 2 + \sum_d (d-2) N_d^M, \qquad d = 2, 4, \ldots \tag{3.19}$$

For a given S-matrix element, the chiral dimension D_L increases with L according to Eq. (3.19). In order to reproduce the (fixed) physical dimension of the amplitude, each loop produces a factor $1/F^2$. Together with the geometric loop factor $(4\pi)^{-2}$, the loop expansion suggests

$$4\pi F \simeq 1.2 \text{ GeV} \tag{3.20}$$

as the natural scale for the chiral expansion [19]. We will have to check whether the coefficients of the local Lagrangian $\mathcal{L}_4$ in (3.15) respect this scale.

It should not come as a surprise that a generic one–loop diagram in CHPT is divergent (non–decoupling EFT are intrinsically non–renormalizable). The divergences can be taken care of once and for all by calculating the divergent part of the one–loop functional. Since it must be a local action with all the symmetries of $\mathcal{L}_2$ [20] and since it has chiral dimension $d = 4$, the corresponding Lagrangian must be of the general form (3.15), albeit with divergent coefficients [5]. The renormalization procedure at the one–loop level is then achieved by simply dividing the coefficients in (3.15) into a (scale–dependent) finite and a divergent part :

$$L_i = L_i^r(\mu) + \Gamma_i \Lambda \;. \tag{3.21}$$

With dimensional regularization, the divergent factor Λ is given by

$$\Lambda = \frac{\mu^{d-4}}{(4\pi)^2} \left\{ \frac{1}{d-4} - \frac{1}{2}[\log 4\pi + 1 + \Gamma'(1)] \right\} \tag{3.22}$$

depending on the arbitrary renormalization scale μ. The relative coefficients Γ_i reproduced in Table 1 are of course chosen in such a way as to cancel the divergences of the one–loop functional. The sum of one–loop amplitudes and of tree amplitudes based on (3.15) is not only finite by construction, but also independent of the scale μ. Again, the scale dependences of the finite one–loop amplitudes and of the renormalized low–energy constants $L_i^r(\mu)$ in the tree contributions cancel. That is all there is to renormalizing a non–renormalizable EFT at the one–loop level !

The final ingredient of $O(p^4)$ is the Wess–Zumino–Witten functional to account for the chiral anomaly. Since I will not need it in the following, let me only emphasize that it has no free parameters, in contrast to the 10 parameters L_i characterizing the "normal" Lagrangian (3.15).

Table 1: Phenomenological values and source for the renormalized coupling constants $L_i^r(M_\rho)$ taken from Ref. [21].

i	$L_i^r(M_\rho) \times 10^3$	source	Γ_i
1	0.7 ± 0.5	$K_{e4}, \pi\pi \to \pi\pi$	$3/32$
2	1.2 ± 0.4	$K_{e4}, \pi\pi \to \pi\pi$	$3/16$
3	-3.6 ± 1.3	$K_{e4}, \pi\pi \to \pi\pi$	0
4	-0.3 ± 0.5	Zweig rule	$1/8$
5	1.4 ± 0.5	$F_K : F_\pi$	$3/8$
6	-0.2 ± 0.3	Zweig rule	$11/144$
7	-0.4 ± 0.2	Gell-Mann-Okubo,L_5, L_8	0
8	0.9 ± 0.3	$M_{K^0} - M_{K^+}, L_5,$	$5/48$
		$(2m_s - m_u - m_d) : (m_d - m_u)$	
9	6.9 ± 0.7	$< r^2 >_{em}^\pi$	$1/4$
10	-5.5 ± 0.7	$\pi \to e\nu\gamma$	$-1/4$
11			$-1/8$
12			$5/24$

3.2 Low–Energy Constants

The phenomenological determination of the constants L_i was pioneered by Gasser and Leutwyler [5]. The present state is summarized in Table 1. If we compare the Lagrangians $\mathcal{L}_2$ and $\mathcal{L}_4$ and recall the chiral expansion parameter

$$\frac{p^2}{(4\pi F)^2} \tag{3.23}$$

suggested by the loop expansion, we expect

$$L_i \lesssim \frac{1}{4(4\pi)^2} = 1.6 \cdot 10^{-3} \ . \tag{3.24}$$

Comparing with the phenomenological values $L_i^r(M_\rho)$ in Table 1, we conclude that this so–called naive chiral power counting is well satisfied in general, with the possible exception of L_3, L_9 and L_{10}.

In principle, the low–energy constants F, B_0, L_i should be calculable in QCD. Awaiting further theoretical progress, we may ask the question whether the $L_i^r(M_\rho)$ can be understood in a more phenomenological way. In the spirit of EFT, we expect all hadronic states that can couple to the pseudoscalar mesons, but which are not included explicitly in the effective Lagrangian, to contribute to the low–energy constants. Looking at the hadronic spectrum, we therefore expect

the meson resonances to play an important rôle for understanding the numerical values of the L_i.

A systematic analysis of the couplings between meson resonances of the type V, A, S, P and the pseudoscalar mesons was performed in Ref. [22] (related work can be found in [23]). We use the general procedure discussed before of coupling matter fields to the Goldstone bosons. For instance, an octet

$$R = \frac{1}{\sqrt{2}} \lambda_a R^a \tag{3.25}$$

of resonance fields transforms as

$$R \xrightarrow{g} h(g, \phi) R h(g, \phi)^{-1} \tag{3.26}$$

where $h(g, \phi)$ is the compensator field in the triplet representation of $SU(3)$. It turns out [22] that for V and A resonances only the octets can contribute to the L_i with the relevant couplings to the pseudoscalar mesons given by

$$
\begin{aligned}
\mathcal{L}_2^V &= \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle \\
\mathcal{L}_2^A &= \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle \\
f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u \ .
\end{aligned}
\tag{3.27}
$$

For S and P, both octets and singlets can contribute.

The coupling constants F_V, G_V and F_A (and the corresponding ones for S, P resonances) can be estimated from resonance decays. At $O(p^4)$, resonance exchange then contributes directly to the L_i, e.g.

$$
\begin{aligned}
L_9^V &= \frac{F_V G_V}{2M_V^2} \ , & L_{10}^A &= \frac{F_A^2}{4M_A^2} \\
M_V &\simeq M_\rho \ , & M_A &\simeq M_{a_1} \ .
\end{aligned}
\tag{3.28}
$$

The results of Ref. [22] can be summarized as follows:

Chiral duality:

> The $L_i^r(M_\rho)$ are practically saturated by resonance exchange. There is very little room left for other contributions.

Chiral VMD:

> Whenever spin-1 resonances can contribute at all ($i = 1, 2, 3, 9, 10$), the $L_i^r(M_\rho)$ are almost completely dominated by V- (and for L_{10} only, also A-) exchange. This also explains the relatively large values of L_3, L_9 and L_{10} in comparison with chiral power counting.

As a practical consequence of chiral duality, all CHPT results to $O(p^4)$ in the meson sector can be recovered in the following way: instead of the $O(p^4)$ Lagrangian (3.15) one can also use the lowest–order resonance Lagrangian [the V, A part is given in (3.27)] and employ a cutoff $\mu \simeq M_\rho$ in the loop integrals. Of course, this alternative approach bears the additional promise of making sense also in higher orders of the chiral expansion (for a practical application of this idea, see Ref. [24]).

Subsequently, it was shown [25] that the high–energy structure of QCD can provide additional information:

- Imposing the QCD constraints at high energies via dispersion relations, all phenomenologically successful models for V, A resonances were shown to be equivalent to $O(p^4)$: tensor field description used in Eq. (3.27), massive Yang-Mills [26], hidden gauge formulations [27], etc.

- With additional QCD–inspired assumptions of high–energy behaviour, like an unsubtracted dispersion relation for the pion form factor, all V and A couplings could be expressed in terms of F_π and M_ρ only. The results are in impressive agreement with the phenomenological values of the $L_i^r(M_\rho)$ for $i = 1, 2, 3, 9, 10$.

There have been several attempts to calculate the L_i "directly" from QCD. The most complete and most advanced analysis by Bijnens, Bruno and de Rafael [28] is based on the extended Nambu–Jona-Lasinio model. The earlier literature on the subject can also be traced back from Ref. [28].

3.3 Recent Applications

Starting with the classical papers of Gasser and Leutwyler [4, 5], CHPT has been applied to a large number of processes to next–to–leading order. Referring to the existing reviews [7–15] for a comprehensive survey of those results, I will briefly mention some recent developments in the mesonic sector.

a. K decays to $O(p^4)$

Inspired by the forthcoming facilities for high–intensity kaon beams like the Φ factory DAΦNE [29] in Frascati, both semileptonic and non–leptonic K decays have been studied extensively. For semileptonic decays, the theoretical situation is very favourable: all low–energy constants appearing in the decay amplitudes are already known (cf. Table 1). Consequently, the CHPT amplitudes are fully determined to $O(p^4)$ and will allow for non–trivial tests of the standard model [21]. The non–leptonic weak interactions require a separate treatment. In particular, a whole new set of low–energy constants appears [30, 31] which are far from being as well determined as the L_i in the strong sector. Fortunately, in rare K decays

relatively few of those constants are relevant. In fact, some of the rare decays like $K_S \to \gamma\gamma$ [32], $K_L \to \pi^0\gamma\gamma$ [33] or $K_S \to \pi^0\pi^0\gamma\gamma$ [34] are completely independent of those constants. Although not as straightforward as in the strong sector, the chiral anomaly also manifests itself in non–leptonic kaon decays [35].

b. Light quark mass ratios

The ratios of light quark masses can be determined to $O(p^4)$ [36] by appealing to Dashen's theorem [37] on the electromagnetic mass shifts of pseudoscalar mesons. Recent investigations [38] have found substantial higher–order corrections to Dashen's theorem leading in particular to a decrease of the ratio m_u/m_d. A welcome consequence is that the CHPT predictions for $\eta \to 3\pi$ decay amplitudes are increased as seems to be required by experiment.

c. CHPT beyond $O(p^4)$

In general, CHPT to $O(p^4)$ has fared quite well in comparison with experiment. However, there are processes where higher–order corrections seem to be larger than suggested by a naive application of chiral dimensional analysis. One such process is $\gamma\gamma \to \pi^0\pi^0$ where the data [39] show an enhancement of the cross section near threshold. This enhancement can be understood via a dispersion theoretical analysis of the scattering amplitude [40]. Very recently, the complete two–loop calculation and estimates of the relevant low–energy constants of $O(p^6)$ have become available [41]. The resulting cross section is in good agreement with experiment.

Another interesting transition is the non–leptonic decay $K_L \to \pi^0\gamma\gamma$ where the experimental spectrum in the invariant mass of the two photons [42] is in very good agreement with the chiral prediction of $O(p^4)$ [33] while the experimental rate seems to be at least twice as large as the theoretical value. The dominant higher–order effects have recently been estimated [43]. Both unitarity corrections and vector meson exchange seem to be required to understand both the rate and the spectrum simultaneously.

4 BARYONS

The construction of chiral invariant Lagrangians with baryon fields is in principle straightforward. As discussed in Sect. 2, there is a well–defined procedure [18] how to couple non–Goldstone degrees of freedom in a chiral invariant manner. All we have to specify is the transformation property of the respective fields under the conserved subgroup.

4.1 Relativistic Formulation

Restricting ourselves to the case of two light flavours, we introduce the familiar isodoublet nucleon field

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix}. \tag{4.1}$$

Under chiral transformations, it transforms as

$$\Psi \xrightarrow{g} \Psi' = h(g, \phi)\Psi \tag{4.2}$$

as discussed in Sect. 2. The covariant derivative

$$\nabla_\mu \Psi = \partial_\mu \Psi + \Gamma_\mu \Psi \tag{4.3}$$
$$\Gamma_\mu = \frac{1}{2}\{u^\dagger[\partial_\mu - i(v_\mu + a_\mu)]u + u[\partial_\mu - i(v_\mu - a_\mu)]u^\dagger\}$$

now includes the external gauge fields v_μ, a_μ.

We now have all the ingredients at our disposal to construct the chiral invariant Lagrangian for the pion–nucleon system

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_M + \mathcal{L}_{MB} . \tag{4.4}$$

As for the purely mesonic Lagrangian $\mathcal{L}_M$ in (3.8), we want to organize the meson–baryon Lagrangian $\mathcal{L}_{MB}$ in a systematic chiral expansion. Here we encounter a basic difference between Goldstone and non–Goldstone fields. Since the nucleon mass remains finite in the chiral limit, the four–momentum of a nucleon can never be "soft". This complicates the chiral counting considerably. For instance, both Ψ and $\nabla_\mu \Psi$ count as fields of $O(1)$, whereas $(i\,\slashed{\nabla} - \overset{\circ}{m})\Psi$ is $O(p)$ like u_μ ($\overset{\circ}{m}$ is the nucleon mass in the chiral limit).

We shall only consider the part of $\mathcal{L}_{MB}$ that is bilinear in the nucleon field. Because of the different Lorentz structure of meson and baryon fields, the chiral expansion of $\mathcal{L}_{MB}$ contains terms of $O(p^n)$ for each positive integer n, unlike in the case of $\mathcal{L}_M$ where the chiral expansion proceeds in steps of two powers of p. In the two–flavour case we have [44]

$$\mathcal{L}_{MB} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \dots \tag{4.5}$$
$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}(i\,\slashed{\nabla} - \overset{\circ}{m} + \frac{\overset{\circ}{g}_A}{2}\,\slashed{u}\gamma_5)\Psi . \tag{4.6}$$

The lowest–order Lagrangian $\mathcal{L}_{\pi N}^{(1)}$ has only two parameters: the nucleon mass $\overset{\circ}{m}$ and the neutron decay constant $\overset{\circ}{g}_A$ (the superscript $\circ$ refers to the chiral $SU(2)$ limit). There are many more parameters in the higher–order Lagrangians $\mathcal{L}_{\pi N}^{(2,3)}$

[44, 45]. With the usual definition of the πN coupling constant $g_{\pi N}$ [44], the Goldberger–Treiman relation [46]

$$\mathring{g}_{\pi N} = \frac{\mathring{m}\mathring{g}_A}{F} \tag{4.7}$$

is an exact relation in the chiral limit. The formalism we have used to construct chiral invariant Lagrangians automatically produces a pseudo–vector πN coupling. The derivative coupling makes a fundamental property of Goldstone bosons manifest: their interactions vanish as the energy goes to zero.

Starting with the work of Gasser, Sainio and Švarc [44], the effective Lagrangian (4.4) has been used to calculate various Green functions and amplitudes with one incoming and one outgoing baryon to one–loop accuracy (see Refs. [12, 14, 47] for recent reviews). There is an intrinsic problem with chiral power counting due to the presence of massive baryon lines in loop diagrams. Referring to Ref. [44] for a detailed analysis, let me try to explain in a qualitative way the difference between purely mesonic CHPT and CHPT with baryons. After the loop integrations and after pulling out a common power of the meson decay constant F, a general CHPT amplitude is a homogeneous function of all the other dimensional variables. Adopting a regularization procedure that does not introduce an extrinsic scale like a momentum cutoff (in a mass–independent renormalization scheme like dimensional regularization with minimal subtraction, the dimensional scale appears only in logarithms), the dimensional variables of a mesonic loop amplitude are precisely the quantities that determine the chiral dimension: external momenta, meson masses and external fields. This leads to formula (3.19) expressing the chiral dimension of the amplitude in terms of the number of loops and of the chiral dimension of vertices. In contrast, baryon propagators introduce the nucleon mass as an additional dimensional quantity, which remains finite in the chiral limit. Consequently, the degree of homogeneity of the baryonic amplitude in question is no more related to its chiral dimension. Therefore, an amplitude with given chiral dimension may receive contributions from diagrams with an arbitrary number of loops [44]. In particular, the coupling constants in the meson–baryon Lagrangian (4.5) get renormalized in every order of the loop expansion. Let us compare once again with meson CHPT: the coupling constant F in the $O(p^2)$ mesonic Lagrangian (3.10) is not renormalized[2] because the divergence of any loop diagram corresponds to a local Lagrangian with chiral dimension $d \geq 4$.

The nucleon mass is comparable to the intrinsic scale $4\pi F \simeq 1.2$ GeV of CHPT. This suggests to set up baryon CHPT in such a way as to expand in

$$\frac{p}{4\pi F} \qquad \text{and} \qquad \frac{p}{\mathring{m}}$$

_____

[2]Remember the constraint on the regularization procedure: a momentum cutoff would lead to a spurious quadratic divergence in F at one loop.

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simultaneously. In the relativistic formulation of CHPT [44], there is an essential difference between F and $\stackrel{\circ}{m}$ in a generic loop amplitude: F appears only in the vertices, whereas the nucleon mass is contained in the propagator.

4.2 Heavy Baryon CHPT

With inspiration from heavy quark effective theory [48], Jenkins and Manohar [49] have reformulated baryon CHPT in precisely such a way as to transfer the nucleon mass from the propagators to the vertices. Although sometimes called the non–relativistic formulation of baryon CHPT, heavy baryon CHPT is formulated in a Lorentz covariant way by defining velocity–dependent fields [3]

$$
\begin{aligned}
N_v(x) &= \exp[i \stackrel{\circ}{m} v \cdot x] P_v^+ \Psi(x) \\
H_v(x) &= \exp[i \stackrel{\circ}{m} v \cdot x] P_v^- \Psi(x) \\
P_v^\pm &= \frac{1}{2}(1\pm \not{v}) , \qquad v^2 = 1 .
\end{aligned}
\tag{4.8}
$$

In the nucleon rest frame $v = (1,0,0,0)$ and N_v, H_v correspond to the usual non–relativistic projections of a Dirac spinor into upper– and lower–component Pauli spinors. In addition, the exponential in Eq. (4.8) is designed to shift the dependence on the nucleon mass. It is instructive to express the free Dirac Lagrangian

$$
\mathcal{L}_0 = \bar{\Psi}(i \not{\partial} - \stackrel{\circ}{m})\Psi
\tag{4.9}
$$

in terms of the velocity–dependent fields N_v, H_v as

$$
\mathcal{L}_0 = \bar{N}_v iv^\mu \partial_\mu N_v - \bar{H}_v(iv^\mu \partial_\mu + 2 \stackrel{\circ}{m})H_v + \bar{N}_v i \not{\partial}^\perp H_v + \bar{H}_v i \not{\partial}^\perp N_v ,
\tag{4.10}
$$

with the transverse Dirac operator $\not{\partial}^\perp$ defined as

$$
\not{\partial} = \not{v} v^\mu \partial_\mu + \not{\partial}^\perp .
\tag{4.11}
$$

Ignoring the "heavy" field H_v for the moment and thereby omitting higher–order corrections in $1/\stackrel{\circ}{m}$, the large–components field N_v obeys the equation of motion

$$
v^\mu \partial_\mu N_v = 0 ,
\tag{4.12}
$$

where the nucleon mass has disappeared by construction. Therefore, the propagator of the field N_v

$$
S(v \cdot k) = \frac{iP_v^+}{v \cdot k + i\varepsilon}
\tag{4.13}
$$

---

[3]Following standard nomenclature, both the external vector matrix field and the four-velocity are denoted by the same symbol v^μ.

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is independent of the nucleon mass, which must have migrated somehow to the vertices of some effective Lagrangian. The momentum k is an off–shell momentum which is on the same footing as the off–shell momentum of a pseudoscalar meson in the propagator (3.16).

Postponing the question what to do with the "heavy" field H_v, let us take a look at chiral power counting for a generic loop diagram in heavy nucleon CHPT. The original meson–baryon Lagrangian (4.4) will be replaced by another effective Lagrangian

$$\mathcal{L}'_{\text{eff}} = \mathcal{L}_M + \mathcal{L}'_{MB} \ , \tag{4.14}$$

where the meson Lagrangian $\mathcal{L}_M$ is unchanged and the meson–baryon Lagrangian $\mathcal{L}'_{MB}$ to be constructed below depends only on N_v, but not on H_v. Of course, it also depends on the meson fields via $u(\phi)$ and on the various external fields. It is again organized in terms with increasing chiral dimensions ($d = 1, 2, 3, \ldots$).

Because of the nucleon propagator (4.13) with chiral dimension -1, the chiral dimension D_L of a general L–loop amplitude with nucleons and pions (more generally, baryons and pseudoscalar mesons) is given by

$$D_L = 4L - 2I_M - I_B + \sum_d d(N_d^M + N_d^{MB}) \ . \tag{4.15}$$

The number of internal meson (baryon) lines is denoted by I_M (I_B) and the numbers of vertices of chiral dimension d are called N_d^M, N_d^{MB} for the two parts of the effective Lagrangian (4.14), respectively. Specializing to the case of a single baryon line running through the diagram[4], we have the relation

$$\sum_d N_d^{MB} = I_B + 1 \tag{4.16}$$

in addition to the general topological relation

$$L = I_M + I_B - \sum_d (N_d^M + N_d^{MB}) + 1 \ , \tag{4.17}$$

valid for any simply connected diagram. Putting everything together, we arrive at the analogue of relation (3.19) in meson–baryon CHPT with one external baryon:

$$D_L = 2L + 1 + \sum_d (d-2)N_d^M + \sum_d (d-1)N_d^{MB} \geq 2L + 1 \ . \tag{4.18}$$

In contrast to the relativistic formulation, chiral power counting in the heavy baryon formulation is completely analogous to the mesonic case. The chiral dimension increases with the number of loops. As for the lowest–order constants F, B_0 in (3.10), the coupling constants in $\mathcal{L}'^{(1)}_{MB} + \mathcal{L}'^{(2)}_{MB}$ are not renormalized in

---

[4]As we shall see shortly, there are no closed baryon loops in this formalism to any finite order in $1/\overset{\circ}{m}$.

any order of the loop expansion, since $D_{L>0} \geq 3$. This applies in particular to the lowest–order constants $\mathring{m}$ and $\mathring{g}_A$.

The most transparent way to construct the effective meson–baryon Lagrangian $\mathcal{L}'_{MB}$ is to start from the path–integral representation of the generating functional of Green functions in the relativistic formulation [44, 50, 51]:

$$e^{iZ[j,\eta,\bar{\eta}]} = N \int [dU d\Psi d\bar{\Psi}] \exp\left[i \left\{ S_M + S_{MB} + \int d^4x (\bar{\eta}\Psi + \bar{\Psi}\eta) \right\} \right]. \qquad (4.19)$$

The action $S_M + S_{MB}$ corresponds to the effective Lagrangian (4.4), the external fields v, a, s, p are denoted collectively as j and $\eta, \bar{\eta}$ are fermionic sources. For two light flavours, the pion–nucleon action is given by

$$S_{MB} = \int d^4x \left\{ \bar{\Psi} \left(i \not{\nabla} - \mathring{m} + \frac{\mathring{g}_A}{2} \not{u}\gamma_5 \right) \Psi + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \ldots \right\} \qquad (4.20)$$

in accordance with Eqs. (4.5),(4.6). In terms of the fields N_v, H_v defined in (4.8), the action takes the form

$$S_{MB} = \int d^4x \{ \bar{N}_v A N_v + \bar{H}_v B N_v + \bar{N}_v \gamma^0 B^\dagger \gamma^0 H_v - \bar{H}_v C H_v \} \qquad (4.21)$$

$$A = P_v^+ \{ iv \cdot \nabla + \mathring{g}_A \, u \cdot S \} P_v^+ + A^{(2)} + A^{(3)} + \ldots \qquad (4.22)$$

$$B = P_v^- \left\{ i \not{\nabla}^\perp - \frac{\mathring{g}_A}{2} u \cdot v \gamma_5 \right\} P_v^+ + B^{(2)} + B^{(3)} + \ldots \qquad (4.23)$$

$$C = P_v^- \{ iv \cdot \nabla + 2 \mathring{m} + \mathring{g}_A \, u \cdot S \} P_v^- + C^{(2)} + C^{(3)} + \ldots \qquad (4.24)$$

$$\not{\nabla}^\perp = \not{\nabla} - \not{v} v \cdot \nabla \, .$$

The terms in A, B, C with chiral dimension $d \geq 2$ are of course due to the Lagrangian $\mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \ldots$. In A and C, the spin operator

$$S^\mu = \frac{i}{2}\gamma_5 \sigma^{\mu\nu} v_\nu \qquad (4.25)$$

appears which obeys the relations [49]

$$S \cdot v = 0 \, , \qquad S^2 = -\frac{3}{4} \, , \qquad \{S^\mu, S^\nu\} = \frac{1}{2}(v^\mu v^\nu - g^{\mu\nu}) \, ,$$

$$[S^\mu, S^\nu] = i\varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma \, .$$

Rewriting also the source term in (4.19) in terms of the fields N_v, H_v with corresponding sources

$$\rho_v = P_v^+ \exp[i \, \mathring{m} \, v \cdot x]\eta \, , \qquad R_v = P_v^- \exp[i \, \mathring{m} \, v \cdot x]\eta \, , \qquad (4.26)$$

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we now integrate out the "heavy" components H_v. Shifting variables

$$H'_v = H_v - C^{-1}BN_v \qquad (4.27)$$

and performing the Gaussian integration over the field H'_v, we arrive at (dropping the sources R_v, $\bar{R}_v$ for simplicity)

$$e^{iZ[j,\rho,\bar{\rho}]} = N \int [dU dN_v d\bar{N}_v] \Delta_H \exp\left[i\left\{S_M + S'_{MB} + \int d^4x(\bar{\rho}_v N_v + \bar{N}_v \rho_v)\right\}\right] \qquad (4.28)$$

where

$$\Delta_H = \exp\left\{\frac{1}{2} \text{ tr } \log C\widehat{C}^{-1}\right\} = \exp\left\{\frac{1}{2} \text{ tr } \log[1 + \widehat{C}^{-1}(iv\cdot\Gamma + \overset{\circ}{g}_A\, u\cdot S + C^{(2)} + \ldots)]\right\} \qquad (4.29)$$

is the corresponding determinant. Expanding the logarithm in (4.29) and taking the trace, one obtains $\log\Delta_H$ as a sum of one–loop functionals with an increasing number of vertices and free H_v propagators $\widehat{C}^{-1}$. On the other hand, the propagator

$$\widehat{C}^{-1}(x) = (iv\cdot\partial + 2\overset{\circ}{m} - i\varepsilon)^{-1} = i\exp[2i\overset{\circ}{m}\, v\cdot x]\Theta(-v\cdot x)\int \frac{d^4k}{(2\pi)^3}\delta(k\cdot v)e^{-ikx} \qquad (4.30)$$

only describes backward propagation along the time–like vector v [50]. This implies that the loops in the expansion of Δ_H can never be closed in coordinate space. Therefore, the trace in (4.29) vanishes and

$$\Delta_H = 1\,. \qquad (4.31)$$

Where have the closed baryon loops gone? Of course, they cannot have disappeared because we have not made any approximations at all by changing variables in the functional integral. The loops are in fact still contained in the **non–local** action

$$S'_{MB} = \int d^4x \bar{N}_v(A + \gamma^0 B^\dagger \gamma^0 C^{-1} B)N_v\,. \qquad (4.32)$$

In the corresponding determinant Δ_N, both the propagators (4.13) (forward propagation along v) and (4.30) appear allowing for non–vanishing traces in coordinate space.

At this point, the crucial approximation of heavy baryon CHPT is made: the non–local part of the functional S'_{MB} is expanded in a series of local functionals with increasing chiral dimensions by expanding C^{-1} in a power series in $1/\overset{\circ}{m}$:

$$C^{-1} = P_v^-\left\{\frac{1}{2\overset{\circ}{m}} - \frac{iv\cdot\nabla + \overset{\circ}{g}_A\, u\cdot S}{(2\overset{\circ}{m})^2} + O(p^2)\right\}P_v^-\,. \qquad (4.33)$$

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At any finite order in $1/\overset{\circ}{m}$, S'_{MB} is a local action with terms of increasing chiral dimensions:

$$
\begin{aligned}
A + \gamma^0 B^\dagger \gamma^0 C^{-1} B = {} & A^{(1)} \\
& + A^{(2)} + \frac{1}{2\overset{\circ}{m}} \gamma^0 B^{(1)\dagger} \gamma^0 B^{(1)} \\
& + A^{(3)} + \frac{1}{2\overset{\circ}{m}}(\gamma^0 B^{(2)\dagger}\gamma^0 B^{(1)} + \gamma^0 B^{(1)\dagger}\gamma^0 B^{(2)}) - \frac{1}{(2\overset{\circ}{m})^2}\gamma^0 B^{(1)\dagger}\gamma^0(iv\cdot\nabla + \overset{\circ}{g}_A \, u\cdot S)B^{(1)} \\
& + O(p^4)
\end{aligned}
\tag{4.34}
$$

with

$$
\begin{aligned}
A^{(1)} &= P_v^+(iv\cdot\nabla + \overset{\circ}{g}_A \, u\cdot S)P_v^+ \tag{4.35} \\
B^{(1)} &= P_v^-\left(i\,\overset{\perp}{\slashed{\nabla}} - \frac{\overset{\circ}{g}_A}{2}u\cdot v\gamma_5\right)P_v^+.
\end{aligned}
$$

At the same time, closed fermion loops have disappeared from heavy baryon CHPT. The only fermion propagator left in the theory is (4.13) which cannot give rise to closed loops.

We have now achieved our goal. The nucleon mass (or rather its chiral limit value) has been transferred from the nucleon propagator to the vertices of a local Lagrangian $\mathcal{L}'_{MB}$ and it appears in exactly the same way as the mesonic constant F. The scene is set for a systematic chiral expansion of baryonic Green functions, simultaneously in $p/4\pi F$ and $p/\overset{\circ}{m}$.

4.3   Neutral Pion Photoproduction at Threshold

Heavy baryon CHPT has been applied extensively (see Refs. [14, 52] for reviews), not only to strong, electromagnetic and semileptonic weak processes, but also to the non–leptonic weak interactions. Even in those cases where the loop calculation had been done previously in the relativistic formulation, it is usually instructive to redo the analysis in the framework of heavy baryon CHPT [51].

A nice example is neutral pion photoproduction at threshold which was analyzed in the relativistic approach by Bernard, Gasser, Kaiser and Meißner [53]. The process to be considered is (see Ref. [54] for a general review of photo– and electroproduction of mesons)

$$
\gamma(k) + p(p_1) \rightarrow \pi^0(q) + p(p_2) \tag{4.36}
$$

and analogously for a neutron target. At threshold only the electric dipole amplitude E_{0+} contributes. It is related to the cross section in the center–of–mass frame through

$$
\lim_{\vec{q}\to 0} \frac{|\vec{k}|}{|\vec{q}|}\frac{d\sigma}{d\Omega} = (E_{0+})^2. \tag{4.37}
$$

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To extract E_{0+} from the amplitude calculated in heavy baryon CHPT, it is convenient to write the scattering amplitude at threshold in the lab frame (nucleon rest frame) as [51]

$$T_{\text{thresh}} = 4\pi i \left(1 + \frac{M_\pi}{m}\right) \chi_2^\dagger \vec{\sigma} \cdot \vec{\varepsilon}(k) \chi_1 E_{0+} \qquad (4.38)$$

in terms of Pauli spinors χ_1, χ_2 and using the Coulomb gauge $\varepsilon^0(k) = 0$.

Let us now calculate the leading–order contribution to T_{thresh} in heavy baryon CHPT. Inspection of the effective Lagrangian $\mathcal{L}'_{MB}$ in Eq. (4.34) shows that the leading term (neither $A^{(1)}$ nor $A^{(2)}$ contribute) is due to the $O(p^2)$ Lagrangian

$$\frac{1}{2 \overset{\circ}{m}} \bar{N}_v \gamma^0 B^{(1)\dagger} \gamma^0 B^{(1)} N_v \qquad (4.39)$$

with $B^{(1)}$ given in Eq. (4.35). The relevant terms in $B^{(1)}$ are obtained with

$$\begin{aligned}
\nabla^\mu &= \partial^\mu + ieA^\mu \frac{1+\tau_3}{2} + \dots \\
u^\mu &= -\frac{1}{F}\partial^\mu \pi^0 \tau_3 + \dots \\
v \cdot A &= 0 \qquad \text{(Coulomb gauge)}
\end{aligned} \qquad (4.40)$$

as

$$B^{(1)} = P_v^-(-e \not{A} \frac{1+\tau_3}{2} + \frac{\overset{\circ}{g}_A}{2F} v \cdot \partial \pi^0 \gamma_5 \tau_3) P_v^+. \qquad (4.41)$$

The scattering amplitude T_{thresh} can easily be read off from Eqs. (4.39) and (4.41):

$$\frac{1}{2 \overset{\circ}{m}} \bar{N}_v \gamma^0 B^{(1)\dagger} \gamma^0 B^{(1)} N_v \quad \hat{=} \quad \frac{-ie \overset{\circ}{g}_A}{4 \overset{\circ}{m} F} v \cdot q \bar{N}_v (\gamma^0 \not{\varepsilon}^\dagger \gamma^0 \gamma_5 + \gamma^0 \gamma_5 \gamma^0 \not{\varepsilon}) N_v =$$

$$= \frac{-ie \overset{\circ}{g}_A}{2 \overset{\circ}{m} F} v \cdot q \bar{N}_v \not{\varepsilon} \gamma_5 N_v = \frac{-ie \overset{\circ}{g}_A}{\overset{\circ}{m} F} M_\pi \bar{N}_v S \cdot \varepsilon N_v \quad (4.42)$$

In the last step, use has been made of the threshold relation $v \cdot q = M_\pi$ and of the identity

$$\bar{N}_v \gamma^\mu \gamma_5 N_v = 2 \bar{N}_v S^\mu N_v . \qquad (4.43)$$

In the lab frame

$$S^0 = 0, \qquad \vec{S} = \frac{1}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

and we recover the structure of Eq. (4.38) with

$$E_{0+}(\gamma p) = \frac{e \overset{\circ}{g}_A}{8\pi F}\left[\frac{M_\pi}{\overset{\circ}{m}} + O(p^2)\right]. \qquad (4.44)$$

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The form of $B^{(1)}$ in (4.41) also shows that $E_{0+}(\gamma n)$ vanishes at threshold in the same leading order in CHPT. The threshold amplitudes are proportional to the electric charges of the targets at leading order in the chiral expansion.

At next–to–leading order, $O(p^3)$, there are both tree–level and loop contributions to T_{thresh}. The obvious candidates for the tree–level amplitudes of $O(p^3)$ are the terms in the third line of Eq. (4.34). However, it turns out that the corresponding amplitude vanishes at threshold [51]. Incidentally, this already tells us that the loop amplitude must be finite because chiral symmetry and gauge invariance do not permit appropriate counterterms of $O(p^3)$.

On the other hand, there are non–vanishing Born diagrams with a nucleon propagator between $O(p^2)$ γNN and πNN couplings, respectively. Those couplings originate both from the term proportional to $\gamma^0 B^{(1)\dagger}\gamma^0 B^{(1)}$ in (4.34) and from the $O(p^2)$ Lagrangian [51]

$$A^{(2)} = \frac{1}{4\,\mathring{m}} P_v^+ \varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma (c_6' f_{+\mu\nu} + c_7' \langle f_{+\mu\nu} \rangle) P_v^+ + \dots \tag{4.45}$$

with $f_{+\mu\nu}$ defined in Eq. (3.27). As is evident from the structure of the Lagrangian (4.45), the constants c_6', c_7' contribute to the Pauli form factor $F_2^V(t)$ of the nucleons. In fact, these constants determine the anomalous magnetic moments of the nucleons in the chiral limit [51]:

$$\mathring{\kappa}_p = \frac{c_6'}{2} + c_7' \,, \qquad\qquad \mathring{\kappa}_n = -\frac{c_6'}{2} + c_7' \,. \tag{4.46}$$

Keeping track of the kinematic factor $1 + M_\pi/m$ in Eq. (4.38), one arrives at the tree level amplitudes

$$E_{0+}^{\text{tree}}(\gamma p) = \frac{eg_A M_\pi}{8\pi F_\pi m} \left[1 - \frac{M_\pi}{2m}(3 + \kappa_p) + O(p^2) \right] \tag{4.47}$$

$$E_{0+}^{\text{tree}}(\gamma n) = \frac{eg_A M_\pi}{8\pi F_\pi m} \left[\frac{M_\pi \kappa_n}{2m} + O(p^2) \right].$$

The alert reader will have noticed that I have used in Eq. (4.47) the physical values g_A, m, F_π, κ_p, κ_n instead of the chiral limit values. That I could do so with impunity, is a major conceptual advantage of heavy baryon CHPT. The relation between the physical and the chiral limit values of all those quantities is such that the differences can be shoved into the higher–order contributions denoted by $O(p^2)$ in (4.47). This is not at all the case in the relativistic formulation for the reasons already discussed. In fact, most of the loop contributions encountered in the relativistic approach renormalize the various constants to their physical values [53]. The final result is the same in both approaches, but the amount of work needed is quite different.

This brings us to the final and most important point. As first shown in Ref. [53], there are in addition to the tree–level amplitudes (4.47) non–trivial loop contributions to the electric dipole amplitudes at threshold at next–to–leading order in the chiral expansion. Once again, it is much easier in the heavy baryon formulation to locate the relevant diagrams [51]. According to chiral power counting as expressed in Eq. (4.18), the one–loop contributions of $O(p^3)$ only involve vertices of the $O(p)$ Lagrangian $A^{(1)}$ in (4.35). Consequently, there is no direct γNN coupling in the Coulomb gauge $v \cdot A = 0$. Moreover, at threshold there is also no direct coupling of the produced π^0 to the nucleon because the coupling strength $S \cdot q$ vanishes at threshold (in the nucleon rest frame, $S^0 = 0$ and $\vec{q} = 0$ at threshold). With these simplifications available in the heavy baryon formulation, out of some 60 diagrams only four non–trivial ones survive, the so–called triangle and rescattering diagrams and their crossed partners [51]. As noticed before, the loop amplitudes are finite and they contribute equally to $E_{0+}(\gamma p)$ and $E_{0+}(\gamma n)$.

The loop amplitudes were omitted in the original derivations [55] and many later rederivations of the low–energy theorems for the electric dipole amplitudes at threshold. The complete low–energy theorems take the form [53]

$$
\begin{aligned}
E_{0+}(\gamma p) &= \frac{e g_A M_\pi}{8\pi F_\pi m} \left[1 - \frac{M_\pi}{2m} \left(3 + \kappa_p + \frac{m^2}{8F_\pi^2} \right) + O(p^2) \right] \qquad (4.48) \\
E_{0+}(\gamma n) &= \frac{e g_A M_\pi}{8\pi F_\pi m} \left[\frac{M_\pi}{2m} \left(\kappa_n - \frac{m^2}{8F_\pi^2} \right) + O(p^2) \right].
\end{aligned}
$$

Although there still seems to be some confusion in the literature [56] as to the status of these results, the low–energy theorems (4.48) are exact predictions of QCD in the limit of isospin conservation. It is a different story that it may be difficult to verify the low–energy theorems experimentally. The relativistic calculation suggests [53, 57] that the chiral expansion for E_{0+} may be slowly converging. Moreover, it may be necessary to include isospin violation [58] to extract the threshold amplitudes from experiment.

Acknowledgements

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